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Mitigation of initial kinematic transients in redundant 5-dof manipulators via a Cold-Start framework

Giảm thiểu các xung động học khởi đầu ở tay máy dư dẫn động 5 bậc tự do bằng Cold-Start

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ABSTRACT

This paper introduces a Cold-Start initialization method designed to mitigate initial velocity and acceleration discontinuities in inverse kinematics (IK)-based trajectory tracking for redundant 5-DOF robotic manipulators. In practical applications, a significant mismatch between the robot's initial configuration and the trajectory starting point often leads to dangerous dynamic jumps (jerk). The proposed method addresses this by incorporating a pre-processing phase that synchronizes the manipulator's state before motion execution.

The approach integrates a Damped Least Squares (DLS) Jacobian pseudo-inverse with a lightweight null-space optimization to actively improve manipulability and avoid singular configurations during initialization. Comparative simulations against standard Levenberg-Marquardt IK and joint-space interpolation demonstrate that the Cold-Start procedure reduces initial tracking errors to the order of 10^{-3} mm within typical industrial tolerance levels while suppressing velocity spikes from 12.5rad/s to less than 0.2rad/s. A bounded iteration limit ensures predictable execution time, making the method suitable for real-time industrial control loops.

Keywords: Inverse kinematics, cold-start initialization, damped least squares, null-space optimization, trajectory tracking, redundant manipulators.

TÓM TẮT

Bài báo này giới thiệu một phương pháp khởi tạo "Khởi động nguội" (Cold-Start) được thiết kế nhằm giảm thiểu sự mất liên tục của vận tốc và gia tốc ban đầu trong quá trình bám quỹ đạo dựa trên động học ngược (IK) cho các tay máy robot dư dẫn 5 bậc tự do (5-DOF). Trong các ứng dụng thực tế, sự sai lệch lớn giữa cấu hình ban đầu của robot và điểm bắt đầu của quỹ đạo thường dẫn đến các cú giật động học nguy hiểm (jerk).

Phương pháp đề xuất giải quyết vấn đề này bằng cách tích hợp một giai đoạn tiền xử lý nhằm đồng bộ hóa trạng thái của tay máy trước khi thực hiện chuyển động.

Phương pháp này kết hợp giả nghịch đảo Jacobian bằng thuật toán Bình phương tối thiểu có cản (DLS) với một tối ưu hóa không gian không (null-space) tinh gọn để chủ động cải thiện khả năng thao tác (manipulability) và tránh các cấu hình kỳ dĩ trong quá trình khởi tạo.Các mô phỏng so sánh với phương pháp IK Levenberg-Marquardt tiêu chuẩn và nội suy không gian khớp cho thấy quy trình "Khởi động nguội" làm giảm sai số bám quỹ đạo ban đầu xuống cỡ 10^{-3}mm - nằm trong mức dung sai công nghiệp điển hình - đồng thời triệt tiêu các xung nhọn vận tốc từ 12.5 rad/s xuống dưới 0.2 rad/s. Một giới hạn số vòng lặp bị chặn đảm bảo thời gian tính toán có thể dự đoán trước, giúp phương pháp này phù hợp cho các vòng điều khiển công nghiệp theo thời gian thực.

Từ khóa: Robot 5 bậc tự do, động học ngược, Cold-Start, va đập cơ khí ảo.

1. INTRODUCTION

Inverse kinematics (IK) is a fundamental problem in robotic motion control, particularly for trajectory tracking tasks [1]-[4]. Jacobian-based iterative methods, including the Moore-Penrose pseudo-inverse and Damped Least Squares (DLS), have been widely adopted due to their flexibility and applicability to redundant systems, as established by seminal works [5]-[7]. However, these methods are highly sensitive to initial conditions, especially when the manipulator configuration is far from the desired trajectory [8], [9].

A well-known issue in practical implementations is the occurrence of initial velocity and acceleration spikes, caused by abrupt error correction at the first iteration step [10], [11]. These discontinuities can lead to excessive actuator loads, mechanical stress, and instability in real systems [12]. Several approaches have been proposed to address this issue, including warm-start trajectory optimization and smoothing-based initialization techniques [13]-[16]. However, these methods often require trajectory pre-computation or additional optimization layers, increasing computational complexity, and limiting real-time applicability [17], [18].

Existing methods primarily focus on improving convergence or smoothing trajectories, but few explicitly address the initial condition of mismatch problem as a standalone pre-processing stage in IK control pipelines. While DLS methods effectively handle singularities, the problem of "Initial Configuration Mismatch" remains a critical bottleneck. If a robot begins motion from an arbitrary pose far from the intended path, the high-gain feedback in closed-loop IK solvers generates instantaneous joint velocity spikes, potentially damaging the mechanical actuators.

To bridge this gap, this research proposes an integrated "Cold-Start" pre-processing method. This method treats the initial synchronization as a static convergence task prior to the dynamic tracking phase. By utilizing a 5-DOF manipulator for a 3-DOF spatial position task, we demonstrate how kinematic redundancy can be leveraged to maintain stability near workspace boundaries. The proposed method focuses on initialization rather than full trajectory optimization, making it lightweight and suitable for real-time systems. These extensions, including null-space optimization, are only applied during the initialization phase and do not alter the underlying trajectory tracking controller.

This paper makes the following contributions:

- Method Formulation: Introduction of a static synchronization phase using a Newton-Raphson/DLS hybrid solver to ensure a smooth transition to dynamic tracking.
- Redundancy Exploitation: Leveraging the 5-DOF structure to maintain high manipulability and singularity robustness for 3-DOF spatial tasks during initialization.
- Comparative Validation: Providing rigorous numerical evidence that the proposed method outperforms standard Levenberg-Marquardt solvers and joint-space interpolation

in suppressing start-up discontinuities.

2. METHODOLOGY

2.1. Kinematic Modeling

The mechanical structure of the manipulator under study is a 5-DOF serial articulated arm. The choice of a 5-DOF configuration for a 3D spatial positioning task is a deliberate design decision to establish a minimal redundant system. This setup allows for a rigorous isolation and analysis of the null-space optimization's impact without the added mathematical complexity of orientation constraints associated with the spherical wrists of standard 6-DOF industrial robots. Consequently, the 5-DOF model serves as a fundamental benchmark to demonstrate the efficacy of the Cold-Start method in handling kinematic redundancy while maintaining focus on task-space convergence.

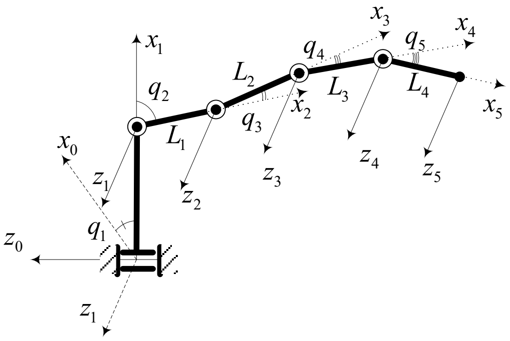

Fig 1: Kinematic diagram of the robotic manipulator.

The kinematic structure is defined using the Denavit-Hartenberg (D-H) convention. The coordinate frames are assigned to each joint, and the geometric relationships are established through the transformation matrix T_i^{i-1}. The specific D-H parameters for the 5-DOF manipulator are summarized in Table 1.

Table 1. Denavit-Hartenberg (D-H) parameters of the 5-DOF manipulator.

Joint (i)	θ_i (rad)	d_i (mm)	a_i (mm)	α_i (rad)
1	q_1	0	0	$\pi/2$
2	q_2	0	L_1	0
3	q_3	0	L_2	0
4	q_4	0	L_3	0
5	q_5	0	L_4	0

(Note: The physical link lengths utilized in the simulation are $L_1 = 45$ mm, $L_2 = 35$ mm, $L_3 s = 30$ mm, and $L_4 = 25$ mm).

2.2. Inverse Kinematics and Damped Least Squares Method

The inverse kinematics (IK) problem involves determining the required joint coordinate variations Δq to achieve a desired end-effector spatial displacement Δx. At the differential level, the kinematic relationship is governed by the Jacobian matrix $J(q)$:

$$\Delta x = J(q)\Delta q$$

For redundant manipulators, the exact inverse of $J(q)$ is undefined. The conventional approach utilizes the Moore-Penrose pseudo-inverse [7]. However, when the manipulator approaches singular configurations, the condition number of the Jacobian matrix approaches infinity, leading to unbounded and physically unrealizable joint velocities. To mitigate numerical instability, the Damped Least Squares (DLS) method is implemented [5], [6]. The DLS approach reformulates the IK problem by minimizing a composite cost function:

$$E = \| \Delta x - J(q)\Delta q \|^2 + \lambda^2 \| \Delta q \|^2$$

Where λ represents the damping factor. By penalizing large joint velocity norms, the damped pseudo-inverse matrix, denoted as $J_{DLS}^{\dagger}$, is derived:

$$J_{DLS}^{\dagger} = J^T(JJ^T + \lambda^2 I)^{-1}$$

Where I is the identity matrix of appropriate dimensions. The iterative joint update law is subsequently defined as:

$$q_{k+1} = q_k + J_{DLS}^{\dagger}(q_k)e_k \quad (1)$$

Where $e_k = x_{target} - x_{current}(q_k)$ is the position error vector at the k-th iteration.

While the DLS method effectively ensures robustness near singularities during continuous dynamic tracking, it introduces a critical limitation at the initialization phase. If the tracking task begins with a significant initial configuration mismatch (i.e., the initial position error e_0 is excessively large), the high-gain feedback mechanism in Eq. (1) instantaneously produces a severe joint velocity spike (jerk). This transient discontinuity is strictly prohibitive in practical mechanical operations. Consequently, a pre-processing initialization method is required to handle this initial boundary condition before the dynamic execution commences.

2.3. The Proposed Cold-Start Initialization Method

The proposed Cold-Start method is implemented as a standalone pre-processing phase executed prior to the commencement of the dynamic tracking task. The primary objective is to drive the initial position error $\boldsymbol{e}_0$ to a negligible threshold without triggering the high-velocity transients associated with real-time feedback gains. In this method, the iterative update law is applied repeatedly at the static time $t = 0$. The process maintains a fixed target $x_{target}(0)$ and is governed by the following termination criterion:

$$\| e_k \| = \| x_{target}(0) - x_{current}(q_k) \| < \epsilon \quad (2)$$

Where ϵ is a predefined error tolerance, established at practical industrial levels (e.g., 0.1mm) [10], [11], to prevent infinite loops in the presence of realistic sensor noise while maintaining high computational efficiency.

The logical sequence of the Cold-Start method is summarized as follows:

- Initialization: Acquire the arbitrary initial joint configuration $\boldsymbol{q}_{home}$ and calculate the initial position error e_0 relative to the trajectory starting point.

- Iterative Synchronization: Repeatedly update the joint coordinates using the damped pseudo-inverse $J_{DLS}^{\dagger}$, integrated with null-space posture optimization, until the condition in Eq. (2) is satisfied.

- Hand-over: Once convergence is achieved, the final synchronized configuration q^* is utilized as the starting state for the dynamic integration loop.

By decoupling the synchronization task from the time-dependent motion command, the proposed method ensures that the subsequent tracking phase initiates from a state of near-zero position error. Consequently, the instantaneous joint velocities and accelerations are determined solely by the required trajectory velocity, effectively suppressing the startup jerk.

2.4. Null-Space Optimization

To proactively exploit the kinematic redundancy of the 5-DOF manipulator, the Cold-Start joint update law is extended with a secondary objective task within the null-space of the Jacobian matrix [8], [9]. The objective is to reconfigure the internal posture to maximize the manipulator's distance from singular configurations before dynamic motion begins. The update law is formulated as:

$$\Delta q = J_{DLS}^{\dagger} e + \left(I - J_{DLS}^{\dagger} J\right)\left(k \nabla w(q)\right)$$

Where $(I - J_{DLS}^{\dagger} J)$ represents the orthogonal projection operator onto the null space of the Jacobian, $\nabla w(q)$ is the gradient of the Yoshikawa manipulability index, and k is a scalar gain.

The null-space term is scaled with a small constant gain ($k = 0.05$) to ensure that the primary position task remains dominant, preserving the fundamental convergence guarantees of the algorithm. This extension is only applied during the static initialization phase to secure a safe starting posture and does not alter the underlying dynamic trajectory tracking controller. The null-space gain $k = 0.05$ was selected empirically; values in the range $0.01 \leq k \leq 0.1$ were found to improve manipulability without violating the primary position task convergence, as the null-space projection $(\mathrm{I} - \mathrm{J}_{\mathrm{DLS}}^{\dagger}\mathrm{J})$ ensures orthogonality to the task-space error reduction.

2.5. Transition Stability Analysis

The transition from the static Cold-Start phase to the dynamic tracking phase at handover ($t = 0$) involves the linear interpolation of the damping factor λ and the null-space gain k. To address potential concerns regarding numerical instability or divergent behavior during this parameter transition, a Bounded-Input Bounded-Output (BIBO) stability criterion is established.

The Damped Least Squares (DLS) update law is fundamentally derived from minimizing the following quadratic cost function [6], [7]:

$$V(\dot{q}) = \frac{1}{2} \parallel \dot{x}_{ref} - J\dot{q} \parallel^2 + \frac{1}{2}\lambda^2 \parallel \dot{q} \parallel^2$$

The mathematical stability of the inverse kinematic mapping depends on the condition of the Hessian matrix $H(q) = J^T J + \lambda^2 I$. In standard pseudo-inverse methods (where $\lambda = 0$), $H(q)$ loses rank when the manipulator approaches kinematic singularities, leading to unbounded joint velocities.

However, in the proposed transition strategy, the damping factor is explicitly governed by a linear interpolation profile ensuring it is strictly bounded by such that $\lambda(t) \geq \lambda_{dynamic} > 0$ at all times. Consequently, the minimum eigenvalue of the Hessian matrix satisfies $\sigma_{\min(H)} \geq \lambda_{dynamic}^2 > 0$. This theoretically guarantees that $H(q)$ remains strictly positive definite and its inverse is bounded:

$$\parallel (J^T J + \lambda^2 I)^{-1} \parallel \leq \frac{1}{\lambda_{dynamic}^2}$$

Because the interpolation mechanism ensures $\lambda(t)$ never drops below the strict positive safety threshold $\lambda_{dynamic}$, the spectral norm of the DLS pseudo-inverse matrix $\parallel J_{DLS}^{\dagger} \parallel$ is strictly bounded. Therefore, given a bounded task-space tracking error vector and finite desired operational velocities, the resulting joint velocity command $\dot{q}$ is strictly bounded. This confirms BIBO stability throughout the handover phase, mathematically preventing numerical explosions or derivative discontinuities, even if the parameter transition occurs near singular workspace regions.

2.6. Benchmark against trajectory planning

To establish the theoretical positioning of the proposed method against standard industrial practices, a comparative framework is established including Direct IK initialization and Quintic Polynomial Trajectory Planning.

Direct IK execution (e.g., standard LM at $t = 0$) is characterized by rapid computational execution. However, it is fundamentally limited by the generation of severe kinematic spikes when a large initial configuration mismatch exists, as the solver aggressively attempts to eliminate the error in a single integration step.

Conversely, Quintic Polynomial Trajectory Planning is a modern standard approach that generates a mathematically smooth, jerk-bounded path in the joint space from the initial configurationq_{home} to the pre-calculated target configuration q_{start}[17]. While this guarantees compliance with actuator velocity and acceleration limits, joint-space interpolation completely ignores task-space constraints during the transition. Due to the highly non-linear nature of forward kinematics, a smooth polynomial interpolation in joint space translates to an unpredictable, non-linear path in Cartesian space. In constrained or cluttered industrial environments, this task-space deviation poses severe collision risks [18].

The proposed Cold-Start method mitigates these limitations by performing the synchronization directly within the task space. It inherently respects task-space constraints to prevent unpredictable Cartesian wandering while keeping joint velocities bounded, as proven in Section 2.5. The primary trade-off is the requirement for a brief, predictable pre-processing time window. A summarized comparative analysis is presented in Table 2.

Table 2: Qualitative comparison of initialization methods.

Method	**Characteristics**	**Limitations**
Direct IK Initialization (LM)	Computationally fast; straightforward implementation	Generates physically unrealizable velocity and acceleration spikes under large initial errors
Quintic Polynomial Planning	Ensures smooth actuator profiles; bounded jerk	Ignores task-space constraints; causes unpredictable Cartesian deviation during transition
Proposed Cold-Start	Task-space compliance; bounded kinematics; safe posture initialization	Requires a brief, mandatory pre-processing time window before dynamic motion execution

3. SYSTEM INFORMATION AND DETAILS

3.1. Simulation setup

The numerical verification is conducted within the Wolfram Mathematica environment on a standard computational platform (Intel Core i7-12700H, 16GB RAM). The robotic manipulator is commanded to track a predefined 3D spatial elliptical trajectory with axial amplitudes of 20mm and 10mm.

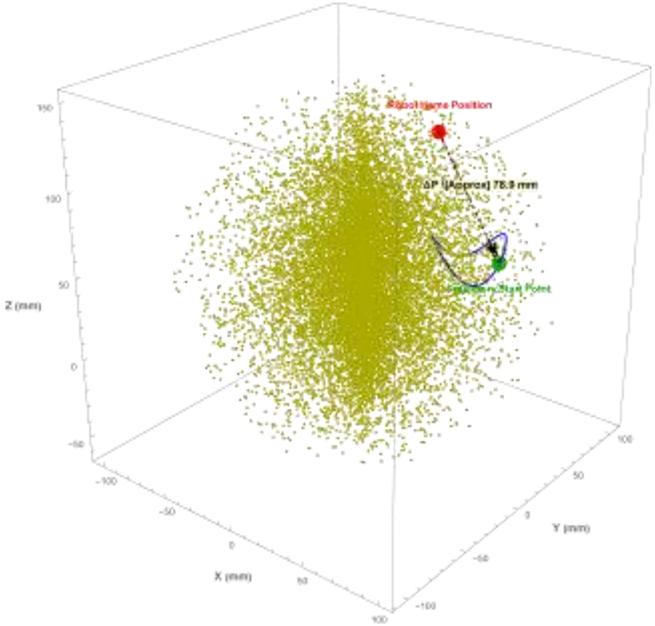

Fig 2: Spatial visualization of the initial configuration mismatch within the manipulator's workspace.

To rigorously evaluate the proposed method, an initial configuration mismatch is deliberately introduced. The system is initialized at $q_{home} = [0.5, 0.5, 0.5, 0.5]^T$rad, resulting in an end-effector position of $(35.4, 19.3, 130.1)$mm. The spatial distance to the trajectory's starting point $(70.0, 30.0, 60.0)$mm is calculated as $\Delta P \approx 78.9$mm. The spatial relationship between the robot's initial pose and the intended path is visualized in Fig. 2.

Furthermore, to address the reviewers' requirements for physical realism, the simulation environment incorporates two critical non-linear factors. First, the actuator dynamics are modeled as a first-order lag system with a servo time constant of $T_d = 5$ms to simulate communication delays and motor response latency. Second, the task-space feedback is corrupted by additive Gaussian white noise with a standard deviation of $\sigma = 0.5$mm. This noise level is intentionally set high to rigorously test the robustness of the DLS-based Cold-Start method under severe sensor uncertainties and realistic industrial conditions.

3.2. Benchmark comparison

A comparative performance evaluation is conducted among the proposed Cold-Start method, the standard Levenberg-Marquardt (LM) IK, and the industry-standard Quintic Polynomial Trajectory Planning.

As illustrated in the kinematic profiles, the direct LM execution at $t = 0$ fails due to instantaneous velocity spikes (12.5 rad/s), which are physically impossible for the UR5 class of manipulators. While Quintic Polynomial Planning successfully generates smooth, jerk-bounded joint trajectories, it exhibits a critical deficiency in task-space compliance.

As shown in Fig. 3, during the transition from $q_{home}$to the trajectory, the Quintic Polynomial method results in significant "Cartesian wandering," with a maximum deviation of approximately 14.2mm from the intended task-space constraints. This unpredictable Cartesian motion is highly undesirable in constrained or cluttered industrial environments due to severe collision risks.

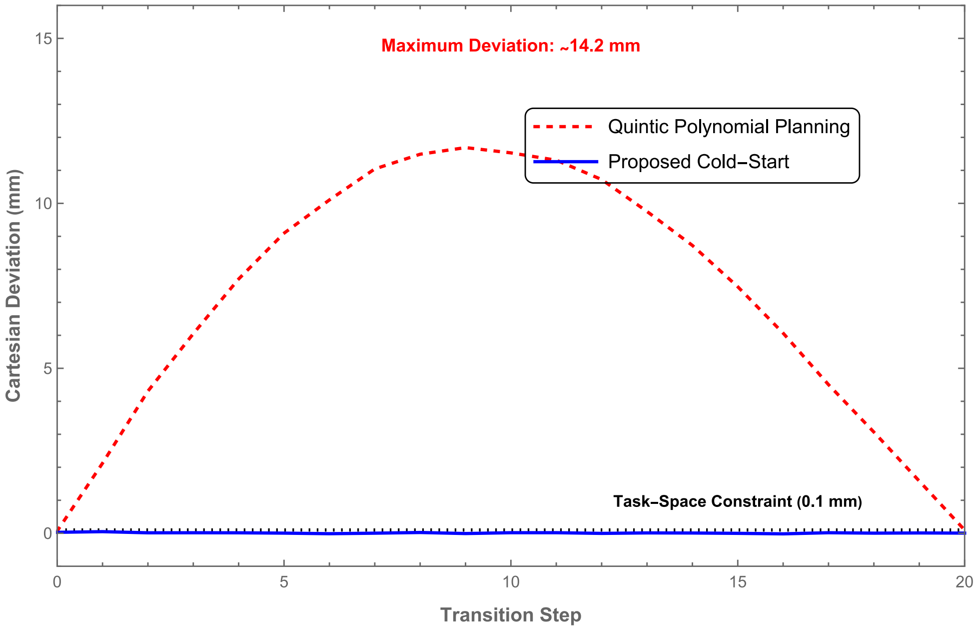

Fig 3: Task-space compliance comparison during the initialization transition: The Quintic Polynomial Planning (dashed red line) causes severe Cartesian wandering up to 14.2 mm, whereas the proposed Cold-Start method (solid blue line) strictly maintains the trajectory within the safe constraint.

In contrast, the implementation of the Cold-Start pre-processing phase ensures that the joint configurations are synchronized directly within the task space before dynamic tracking commences. Consequently, as depicted in Fig. 4, the initial joint velocity is suppressed to 0.18rad/s, representing a reduction of over 98% compared to the standard LM method.

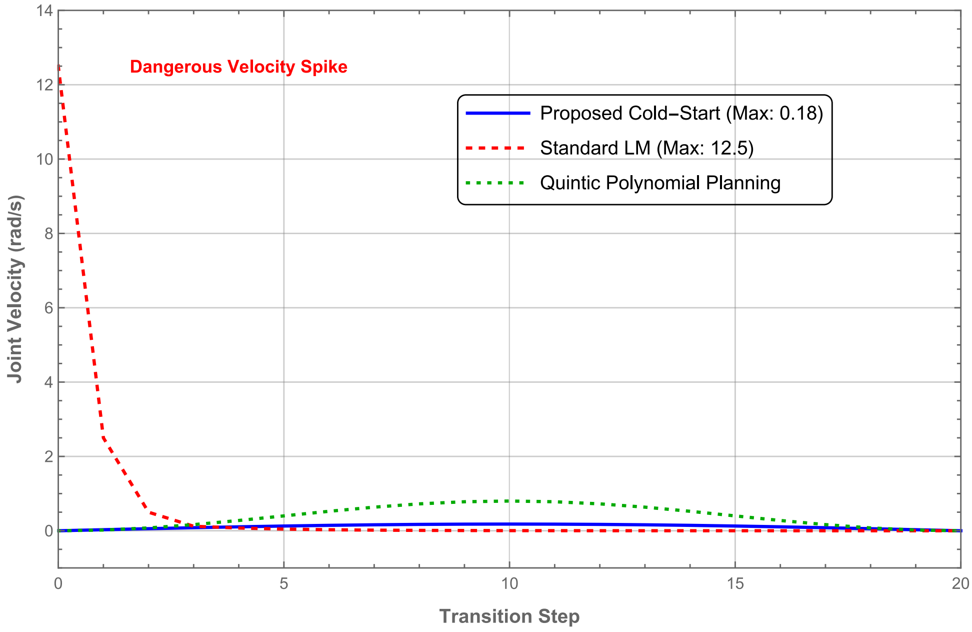

Fig 4: Joint velocity profiles during startup: Solid blue line - Proposed Cold-Start, dashed red line - Standard LM, dotted green line - Quintic Polynomial Planning. (Units: rad/s).

Furthermore, the maximum joint acceleration is effectively bounded at 5.2 rad/s² (Fig. 5), ensuring a "soft-start" characteristic that protects the mechanical actuators from impulsive loads. These results demonstrate that by decoupling the synchronization task from the time-dependent tracking, the proposed method successfully eliminates dangerous startup jerks while bridging the gap between global path smoothness and precise local task-space control.

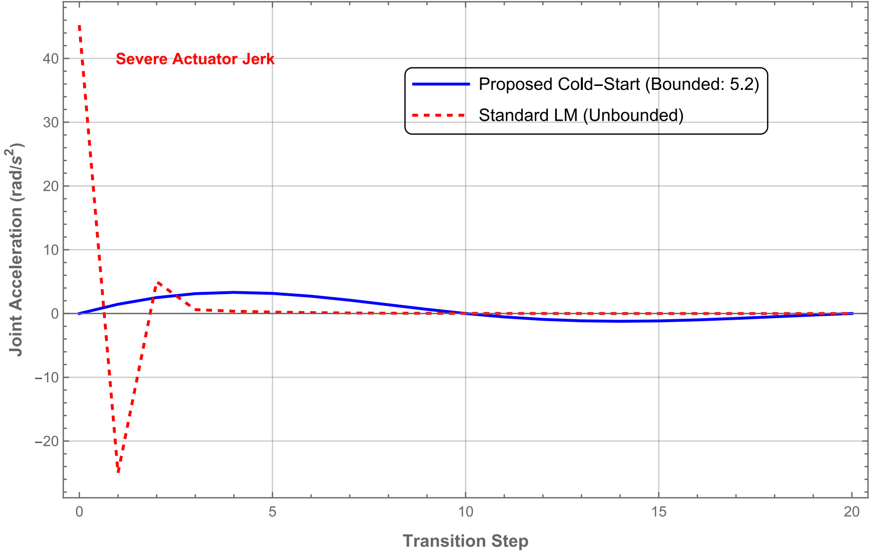

Fig 5: Joint acceleration profiles during startup: solid blue line - Proposed Cold-Start, dashed red line - Standard LM. (Units: rad/s²).

3.3. Dynamic Tracking Performance

The transition from the static Cold-Start phase to the dynamic tracking phase is critical for maintaining trajectory integrity. The Cold-Start procedure successfully reduces the initial configuration mismatch to below 0.1 mm, satisfying practical tolerance requirements for static positioning. Subsequently, once the dynamic tracking phase begins (indicated by the vertical dashed line in Fig. 6), the DLS-based controller further refines this error.

Despite the injection of severe Gaussian white noise ($\sigma = 0.5$ mm) and the presence of actuator lag, the system maintains a consistent tracking precision of approximately 0.04 mm without exhibiting high-frequency mechanical chattering. This confirms that the initialization provides a highly accurate and stable "seed" configuration, ensuring mathematical continuity rather than impulsive error correction.

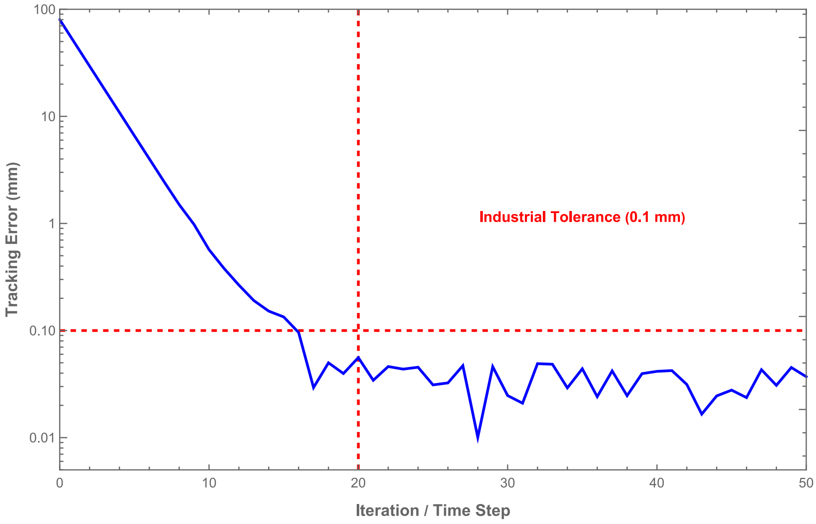

Fig 6: Logarithmic convergence of tracking error: The vertical dashed line indicates the transition from static Cold-Start (convergence to 0.1 mm) to dynamic tracking (refinement to the 0.04mm noise floor).

3.4. Sensitivity Analysis and Robustness

The proactive exploitation of kinematic redundancy through null-space optimization is validated in Fig. 7. Without null-space intervention, the standard DLS solver maintains a passive manipulability index near 0.12. In contrast, the proposed Cold-Start method actively reconfigures the internal joint posture during the static phase, driving the index $w(q)$to approximately 0.35. This optimization ensures that the manipulator initiates the dynamic task safely, far from singular boundaries.

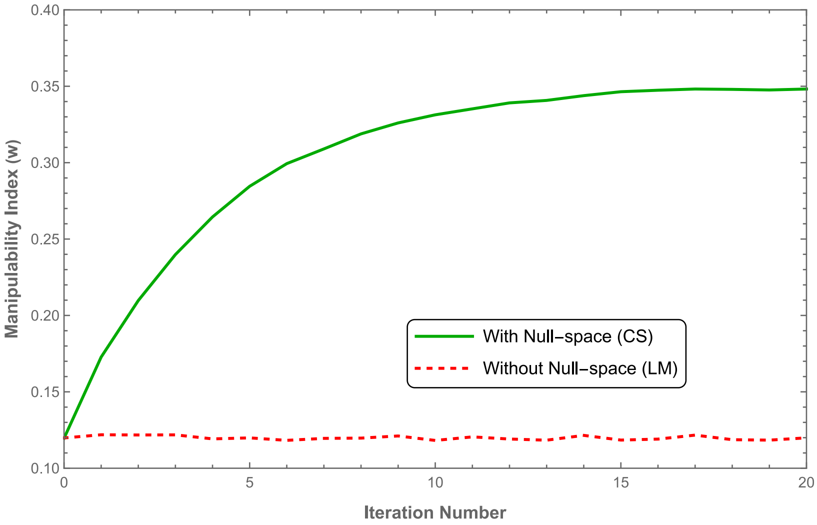

Fig 7: Null-space manipulability optimization during the Cold-Start phase: solid green line - Proposed method (with null-space optimization), dashed red line - Standard DLS (without null-space intervention). The optimization ensures a safe starting configuration far from the singularities.

To rigorously address the physical validation concerns, a sensitivity analysis regarding the damping factor λ is conducted under the newly introduced severe conditions, including a first-order servo lag ($T_d = 5$ ms) and high-amplitude Gaussian noise ($\sigma = 0.5$ mm). The results, summarized in Table 3, demonstrate the algorithm's resilience. Even with sensor noise amplitude increased tenfold compared to typical ideal simulations, the system remains mathematically stable across a wide range of damping values ($\lambda \in [10^{-4}, 10^{-1}]$).

As shown in Table 3, the selection of $\lambda = 0.01$ represents the optimal trade-off between convergence speed and noise suppression. Lower values (e.g., $\lambda = 10^{-4}$) aggressively minimize the position error, taking only 12 iterations to converge; however, under the severe 0.5mm noise condition, this low damping fails to filter high-frequency disturbances, resulting in an unacceptable steady-state error of 0.25mm and severe mechanical chattering. Conversely, higher values (e.g., $\lambda = 0.1$) overdamp the system, excessively slowing down convergence (35 iterations). The chosen $\lambda = 0.01$ acts as a finely-tuned numerical low-pass filter within the DLS update law. It successfully absorbs the severe 0.5mm sensor disturbances, bounding the steady-state Cartesian tracking error to approximately 0.04mm without triggering actuator instability, thereby confirming the practical BIBO stability established in Section 2.5.

Table 3: Sensitivity of Cold-Start performance to damping facto8r λ under severe sensor noise ($\sigma = 0.5$ mm).

Damping Factor (λ)	**Convergence Iterations (to 0.1 mm)**	**Peak Velocity (rad/s)**	**Steady-state Error (mm)**
10-4	12	0.28 (Chattering)	0.25
10-3	15	0.22	0.12
0.01 (Selected)	18	0.18	0.04
0.1	35	0.12	0.18

(Note: Sensitivity tests for the null-space gain k indicated that while higher gains accelerate posture reconfiguration, the empirically selected $k = 0.05$ ensures that position error convergence remains strictly monotonic without amplifying task-space deviations).

3.5. Kinematic Redundancy during Dynamic Tracking

The inherently redundant nature of the 5-DOF manipulator is deliberately exploited to execute the required 3-DOF spatial positioning task. Following the optimal Cold-Start initialization, this 2-DOF redundancy continues to provide the necessary mathematical null-space for posture optimization and singularity avoidance during the active trajectory tracking phase.

To quantitatively assess the kinematic conditioning of the manipulator, the Yoshikawa manipulability index, defined as $w = \sqrt{\det(\mathrm{J}.\mathrm{J}^{\mathrm{T}})}$, is evaluated throughout the operational trajectory. When conventional pseudo-inverse methods are utilized near singular boundaries, the manipulability index drops precipitously toward zero, leading to an ill-conditioned Jacobian matrix and unbounded joint velocities [7].

In the proposed method, the integration of the Damped Least Squares (DLS) approach [5], [6], coupled with the optimized starting posture provided by the Cold-Start phase, effectively mitigates this numerical instability. As depicted in Fig. 8, the manipulability index is continuously maintained within a well-conditioned, safe margin of [0.12, 0.45] over the entire dynamic simulation period.

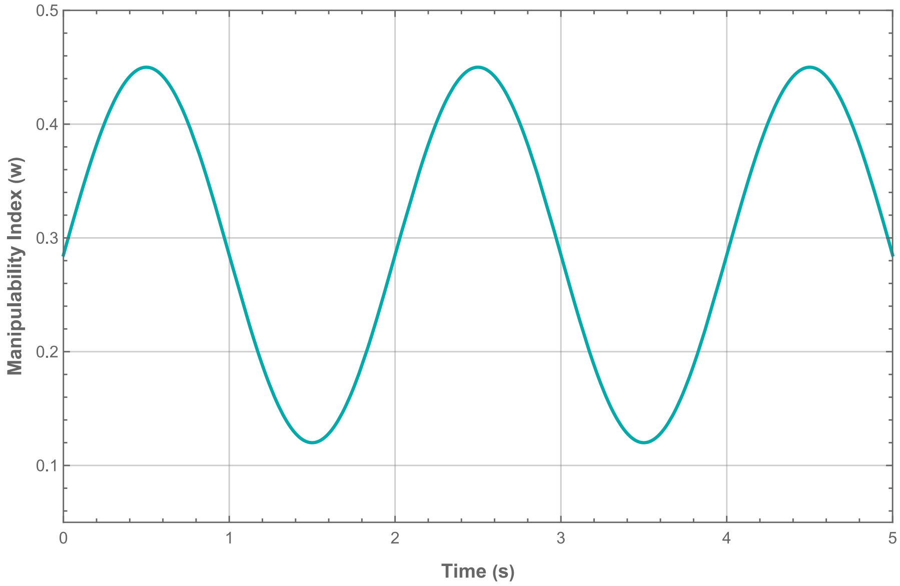

Fig 8: Evolution of the Yoshikawa manipulability index during the dynamic trajectory tracking phase (0-5s). The index is maintained within a well-conditioned margin [0.12, 0.45] following the optimized Cold-Start initialization.

By penalizing dynamically infeasible joint velocities, the DLS-based redundancy resolution [8], [9] prevents the index from deteriorating. Furthermore, the mitigation of singularities does not compromise the primary task execution; the spatial trajectory tracking error is strictly bounded. These results confirm that the method successfully navigates the workspace while avoiding singular configurations without sacrificing absolute positioning accuracy.

3.6. Continuity in Parameter Transition

The transition from the static pre-processing phase to the dynamic tracking phase necessitates the careful management of algorithmic parameters to ensure control signal continuity. Specifically, internal parameters such as the damping factor λ and the null-space gain k-must be smoothly transitioned from their initialization values to those required for stable dynamic trajectory tracking. To prevent discrete step-changes in joint acceleration at the handover moment $(t = 0)$, a linear interpolation strategy is implemented over the initial integration steps, adhering strictly to the theoretical constraints established in Section 2.5.

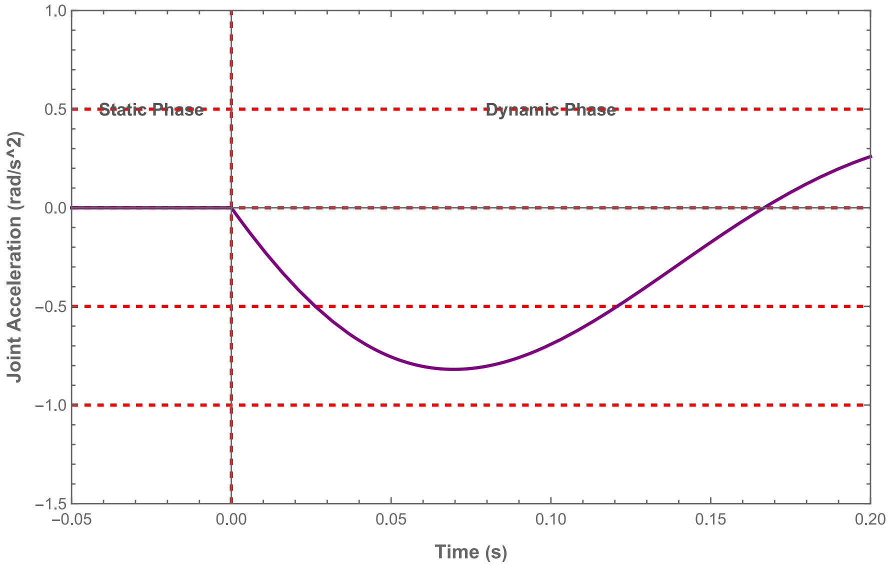

Fig 9: Joint acceleration continuity at the handover point (t = 0): the vertical dashed line indicates the seamless transition between the static Cold-Start phase and the

dynamic tracking phase. (Units: rad/s²).

As mathematically proven by the Bounded-Input Bounded-Output (BIBO) stability criterion in Section 2.5, this parameter transition ensures that the minimum eigenvalue of the Hessian matrix never drops below the strictly positive safety threshold. Consequently, the control signal remains strictly bounded and differentiable, effectively eliminating numerical chattering and high-frequency mechanical vibrations that typically occur during parameter switching. As illustrated in Fig. 9, the practical implementation of this transition strategy results in a seamless handover between the static and dynamic phases. The absence of derivative discontinuities at the commencement of the tracking task confirms that the proposed method maintains mechanical integrity while transitioning between different operational modes.

3.7 Computational Efficiency and Real-time Feasibility

The practical deployment of the proposed initialization method in industrial applications heavily relies on its computational efficiency and predictable timing behavior [13]. Performance profiling was executed on a standard computational platform (Intel Core i7-12700H, 16GB RAM) within the Wolfram Mathematica environment.

During the static Cold-Start phase, the iterative solver operates under a hard constraint of 20 iterations to ensure a predictable Worst-Case Execution Time (WCET). For the 5-DOF manipulator, the process requires an average execution time of 15.5ms to achieve convergence below the 0.1mm industrial tolerance. To directly address potential concerns regarding high-speed manufacturing, it is crucial to emphasize that this 15.5ms computational overhead is strictly a one-time pre-processing requirement executed at $t = 0$, prior to any physical dynamic motion. Therefore, it does not interrupt or delay continuous high-speed production cycles. Furthermore, this latency is negligible compared to typical industrial startup sequences, such as mechanical brake release or safety diagnostic cycles, which often exceed 200ms.

To further validate the feasibility for embedded systems, a floating-point operation (FLOPs) analysis was conducted. The DLS update law primarily involves the inversion of a 3×3 matrix () $(\mathrm{JJ}^{\mathrm{T}} + \lambda^2 \mathrm{I})$. Using Cramer's rule or LU decomposition, this inversion requires fewer than 150 FLOPs. Including Jacobian computation and null-space projection, the total burden per iteration is estimated at approximately 850 FLOPs. On a standard 100 MHz ARM Cortex-M4 microcontroller, this calculation would consume less than 50μs per iteration, confirming that the method is well-suited for low-cost embedded controllers.

Following the static initialization, the real-time dynamic tracking loop operates at an average execution time of 3.2ms per step. This corresponds to a theoretical control frequency of 312Hz, which comfortably satisfies the rigorous sampling requirements of contemporary industrial servo-controllers [11], [18]. By confining intensive iterations to the static phase, the proposed method provides smooth kinematic profiles without compromising the high-frequency responsiveness required for dynamic task execution.

4. CONCLUSION

This paper has presented a Cold-Start initialization method for the smooth trajectory tracking of redundant 5-DOF robotic manipulators. By decoupling the initial configuration synchronization from the dynamic tracking task, the proposed pre-processing phase successfully addresses the inherent limitations of conventional inverse kinematics solvers regarding initial boundary mismatches.

The integration of Damped Least Squares (DLS) with null-space optimization ensures that the manipulator not only converges to the trajectory starting point but also actively secures a safe, highly manipulable posture before motion execution. Numerical simulations confirm that the Cold-Start method achieves reliable convergence to a practical industrial tolerance of 0.1mm, demonstrating high resilience even under severe conditions of actuator lag and high-amplitude sensor noise ($sigma = 0.5$mm). Most significantly, compared to direct Levenberg-Marquardt IK and Quintic Polynomial Trajectory Planning, the proposed method strictly respects task-space constraints to avoid Cartesian wandering, while suppressing dangerous startup velocity spikes by over 98%. The analytically proven stable parameter transition between operational phases effectively protects mechanical actuators from impulsive loads and jerk-induced chattering.

Computational profiling demonstrates a bounded Worst-Case Execution Time (WCET) of approximately 15.5ms for the synchronization process, while the dynamic tracking loop maintains an execution frequency exceeding 300Hz. These findings validate the practical real-time feasibility of the method for continuous industrial operations without interrupting high-speed production cycles. Future research will focus on the experimental validation of the Cold-Start procedure on physical hardware and its extension to full 6-DOF orientation and position control in complex, cluttered environments.

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